The transmission of monetary shocks in production networks

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Abstract

This paper proposes a theoretical framework to study the interest rate channel and asset price channel of monetary transmission in a multisector RBC setting. The model highlights how money supply shocks propagate through trade in the production and investment networks, both *upstream* due to household wealth effects and the cash-in-advance assumption, and *downstream* due to the interest rate channel and Tobin's Q channel. In both cases, input-output linkages in the intersectoral trade of materials and capital are critical for the propagation of monetary shocks. An application to the 2005-2018 US input-output tables estimates that 52%-59% of the total effect of money supply shocks on production can be attributed to network effects. Most of this result is due to material linkages, with capital linkages representing approximately one-fifth of the effects. Downstream propagation is estimated to be almost 2.6 times bigger than upstream propagation.

Keywords: Monetary transmission, input-output linkages, production network, investment network

1 Introduction

Networks have recently gained popularity in macroeconomics for their potential to explain how sectoral shocks can cause aggregate fluctuations. Indeed, business cycle models can be enriched with insights from graph theory to better understand the propagation of shocks through the complex web of inputs and outputs that forms the economy. One important area that benefits from this new methodology is the study of monetary transmission.

This paper formalises a network model of production and sectoral investment to study the role of input-output linkages in the monetary transmission mechanism. By adding the possibility for firms to invest intermediate inputs and accumulate capital to the multisectoral production network model of Acemoglu et al. (2012), monetary transmission can be modelled as follows. Using the cash-in-advance assumption, increases in money supply are treated as demand-side shocks that decrease the interest rate and increase firms profits, resulting in higher equity prices. First the decrease in interest rate results in a lower cost of capital, which has an expansionary effect on firms investments (*interest rate channel*). Second, the increase in asset prices has an expansionary effect on firm investments via the Tobin's Q channel and an expansionary effect on consumer expenditures via household wealth effects (*asset price channel*).

Modifying the baseline network model to account for these channels allows to study the propagation of money supply shocks across sectors. Indeed, firms facing a large shock will modify

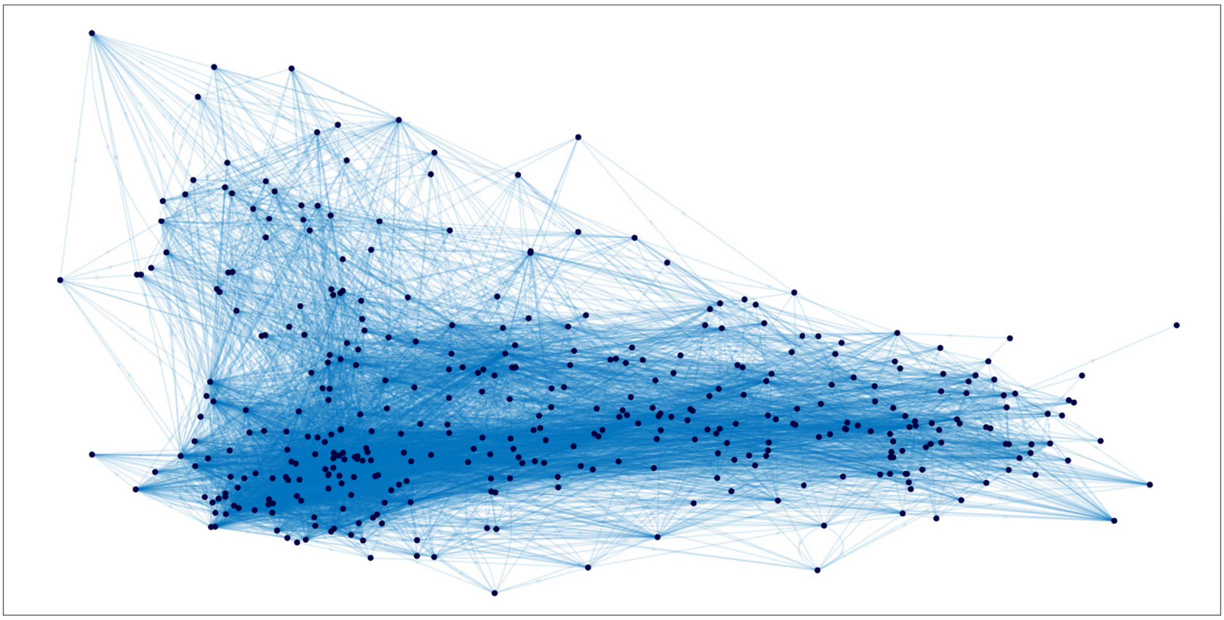

Figure 1: US production network of 392 sectors in 2012. *Notes:* Directed weighted graph using the US input-output tables data from the Bureau of Economic Analysis. Each vertex represents a sector and each edge represents a directed trade relation with another sector. Edges below 200 million dollars per year and self-loops are omitted.

their demand and supply of intermediate goods, so that the shock propagates to other industries through trade. While demand and supply shocks usually propagate either upstream or downstream (Acemoglu et al., 2016), the modified model shows that money supply shocks can propagate in both directions. They propagate upstream because of the wealth effect channel and the cash-in-advance assumption, which make the shocks propagate from the household to its suppliers, which then impact their own suppliers of intermediate goods, and so on. They also propagate downstream because of the interest rate channel and Tobin's Q channel, which decrease the costs of investments for firms and make the shocks propagate from the firms to their intermediate customers, which then impact their own customers, and so on to the household. This cascade of shocks through intersectoral trade is known as the network effect or indirect effect. These network propagation patterns play a key role in the transmission of monetary shocks as they not only affect the firms, but also impact their clients and suppliers, the clients and suppliers of its clients and suppliers, and so on through trade. The upstream propagation is shown in theorem 1, and the downstream propagation in theorem 2.

An empirical analysis is conducted using quarterly US data from 2005 to 2018. Using autoregressions of sectoral output on money supply shocks, between 53% and 59% of the total effect of the shocks can be attributed to network effects depending on which reference year is chosen for the input-output matrix. Approximately one-fifth of the effect is due to the investment network and its capital linkages, while most of the effect is carried by the production network and its materials linkages. As such, input-output linkages and the network structure of production are empirically important mechanisms for the propagation of monetary shocks. Moreover, the role of network effects in the propagation of monetary shocks varies significantly between sectors, highlighting an important sectoral heterogeneity in monetary transmission. The analysis also outlines that the input-output matrices are rather stable in the short-run, and that a CES production function or flexible price framework may be more adequate than Cobb-Douglas production functions due to some negative network effects. Lastly, downstream propagation is estimated to be 2.6 times bigger than upstream propagation, highlighting the importance of investments and suppliers-to-clients propagation in the transmission of monetary shocks. The short-run stability of input-output matrices is presented in table 1, full results using the 2018 input-output linkages are presented in table 2, the average network effects for multiple reference years are presented in table 3, and the upstream/downstream estimation is in table 4.

In a more general context, networks can be used to model large and complex systems of agents (vertices) and the relations between them (edges). They are based on the mathematical field of graph theory and can be used to model many types of systems such as infrastructure networks, social networks, supply chain networks, or cyber networks. In macroeconomics, they are mainly used to model the trade relationships between firms. Indeed, intersectoral trade creates a large web of direct and indirect relations across industries that can be observed empirically via input-output tables (figure 1). The interconnectedness observed in input-output linkages means that idiosyncratic shocks have the potential to propagate through trade and create aggregate fluctuations (Acemoglu et al., 2012; Carvalho, 2014). These network effects are even more important when accounting for the intersectoral capital investments of firms, as it creates another layer of trade (Vom Lehn and Winberry, 2022). They also have important effects in the propagation of shocks across countries, such as with international inflation spillovers (Auer et al., 2019). Other studies have argued that the network structure of the economy has important implications for industrialisation, long-run growth, and monetary policy (Carvalho and Tahbaz-Salehi, 2019). Even if a very recent literature started to use firm-level data[1], this paper and most of the literature use sector-level data due to the scarcity of granular datasets.

The presence of input-output linkages is an important consideration for the conduct of monetary policy. By identifying which industries are the largest and stickiest input suppliers, optimal policies can assign larger weights to their prices and better control inflation, leading to welfare gains (La'O and Tahbaz-Salehi, 2022; Rubbo, 2020). Moreover, it is a common feature of multi-sectoral models to increase the degree of monetary non-neutrality, which explains a larger part of business cycles fluctuations (Nakamura and Steinsson, 2010). The monetary transmission mechanism is also affected by the network structure of the economy. Production networks increase the propagation of price rigidity from input suppliers to other industries, which amplify the effect of monetary shocks (Ghassibe, 2021b). Additionally, input-output linkages have been shown to interact with heterogeneity in price stickiness and consumption share to determine which sectors are the most important in monetary transmission (Pasten et al., 2020). The endogenous formation of input-output linkages also provides new sources of nominal rigidities, rationalising non-linearities in monetary transmission (Ghassibe, 2021a). Lastly, network ef-

[1] See for instance Carvalho et al. (2021) which uses Japanese data and Bernard et al. (2022) which uses Belgian data.

fects account for a large portion of the transmission of monetary shocks to equity prices, both for domestic assets (Ozdagli and Weber, 2017) and for foreign assets via international spillovers (Di Giovanni and Hale, 2022). This paper contributes to this growing literature by providing a theoretical framework to study the monetary transmission mechanism with networks, as well as providing new insights for the role of input-output linkages in monetary transmission based on the interest rate and asset price channels.

The paper is organised as follows. Section 2 discusses how to implement the channels of monetary transmission in production networks and defines the modified model. Section 3 analyses the propagation of money supply shocks in the model and the theoretical role of input-output linkages. In section 4 an empirical analysis is conducted to estimate the share of network effects in the transmission of money supply shocks. Section 5 concludes.

2 A network model of investments and production

This section lays out the theoretical foundations of the model, discusses the elements that need to be added for monetary transmission, and presents the competitive equilibrium and network structure of the model.

2.1 Combining networks and monetary transmission

The network structure of production is an important consideration for monetary transmission due to the heterogeneity in how sectors react to monetary shocks. Focusing on the asset price channel, the sensitivity of firm equity prices to money supply shocks varies significantly based on debt, cash-flows, size (Ehrmann and Fratzscher, 2004), and financial constraints (Ottonello and Winberry, 2020). More broadly, there is a significant heterogeneity in the sensitivity of production, consumption, and other variables to monetary shocks (Kaplan and Violante, 2018). All of these elements create heterogeneity between industries, calling for a multisector model of production instead of a representative agent model. The advantage of the network methodology is then to provide a simpler framework to analyse the intersectoral linkages and study the propagation mechanisms of shocks (Carvalho and Tahbaz-Salehi, 2019).

Recent research took two distinct approaches in combining networks and monetary policy. The first approach is to use the business cycle production network model of Acemoglu et al. (2012), which is a variation of the multisector RBC model of Long and Plosser (1983). This approach is the most widespread in the macroeconomic networks literature and has the advantage of providing a clear framework for input-output linkages, firms dynamics, and propagation mechanisms. This business cycle approach is used for monetary policy by Ozdagli and Weber (2017); Di Giovanni and Hale (2022); La'O and Tahbaz-Salehi (2022). The second approach is to use a multisector New Keynesian model, which has the advantage of clearly defining inflation and central banks behaviour. This framework is less common in the networks literature as it only has some advantages for monetary topics and misses some of the shock propagation mechanisms. This New Keynesian approach is used by Ghassibe (2021b); Rubbo (2020); Pasten et al. (2020).

This paper uses the business cycle approach to model monetary transmission. Even if the New Keynesian approach is arguably better to study optimal monetary policy and the role of price rigidity in monetary transmission, the business cycle approach offers a more thorough analysis of network effects and propagation mechanisms. Moreover, Ghassibe (2021b) already offers a convincing monetary transmission model and empirical network analysis in the New Keynesian framework. On the other hand, research on monetary transmission in the business cycle framework is lacking. Ozdagli and Weber (2017) and Di Giovanni and Hale (2022) use this approach to empirically study the transmission of money supply shocks to asset prices and estimate that between 50-80% of the transmission could be attributed to network effects. While their estimates are convincing, they only focus on the transmission of shocks to asset prices, and not on the whole monetary transmission process. Moreover, they do not take capital accumulation into consideration, even though investments are an important mechanism of monetary transmission. As such, there exists a gap in the literature when it comes to modelling the full monetary transmission process with the network business cycle model.

In order to model monetary transmission, several elements need to be added to the baseline business cycle production network model. First, firms need to be able to accumulate capital, as investments are a key part of multiple monetary transmission channels. While capital accumulation has been formalised in a handful of networks studies[2], the vast majority of the macro networks literature uses only labour and intermediate inputs as factors of production. This can be explained by the focus of many studies on static and short-run effects, as well as by the scarcity of capital investment input-output data. Nevertheless, capital linkages are an important part of the propagation of shocks and not including them would underestimate the network effects (Atalay, 2017). Second, the model needs to account for the role of the interest rate in the transmission of shocks. This feature is uncommon in production networks and has only been used in models based on the New Keynesian framework. Third, households need to own the firms and receive their profits to account for household wealth effects. This is already a part of multiple production networks models and is straightforward to implement.

The monetary transmission mechanism needs to be laid out to understand why these elements are needed in the production network model. Monetary transmission refers to the channels through which monetary policies have an effect on the economy. The literature commonly refers to four main mechanisms: (i) *the interest rate channel*, where money supply shocks decrease the interest rate, which then decreases the costs of capital and has an expansionary effect on firms investments and households durable expenditures; (ii) *the exchange rate channel*, where money supply shocks decrease the interest rate, which then decreases the exchange rate, which then has an expansionary effect on net exports; (iii) *the asset price channel*, where money supply shocks increase equity prices, which then have an expansionary effect on firms invest-

[2]See Foerster et al. (2011) for an early attempt at capital accumulation, Atalay (2017) for the role of capital linkages in the amplification of micro shocks, and Vom Lehn and Winberry (2022) for the recent regain in interest

ments and household consumption; and (iv) *the credit channel*, where money supply shocks affect credit markets in a variety of ways, which then have an expansionary effect on firms investments and household consumption (Mishkin, 1995). The main transmission mechanisms studied in this paper are the interest rate channel and the asset price channel.

The asset price transmission mechanism of a money supply shock can be summarised as follows, based on Mishkin (2001) and Di Giovanni and Hale (2022). An expansionary money supply policy directly increases the households demand for final goods. This increases their prices, which then increase the firms profits and the demand of intermediate inputs from firms. The higher profits lead to higher equity returns, which then increase the firms equity value. The increase in market value then has two effects according to the asset price channel. First, it lowers the cost of capital acquisition by Tobin's Q theory, which increases firms investments. Second, it has wealth effects for households since they own the firms equity, which increases household expenditures.

The increased market value of firms also affects the credit channel in two ways. First, it decreases the risk of moral hazard and adverse selection for firms by the balance sheet channel, which increases firms investments. Second, there are household liquidity effects, as the probability of financial distress decreases for households, which increases their expenditures.

This paper focuses on the interest rate channel and the asset price channel (itself composed of Tobin's Q channel and household wealth effects). It does not include the credit channel. Most of the credit channel effects would require additional modelling elements that would increase the complexity of the model. Moreover, the size of the credit channel is negligible compared to the interest rate and asset price channels, and its role in the monetary transmission mechanism has often been described as empirically insignificant (Ramey, 1993). The paper also does not include the exchange rate channel. While spillovers and other international elements are an important part of monetary transmission, switching to an open model or including exchange rates in any other way would once again make the model more complex than it aims to be. For these reasons, the credit channel and exchange rate channels are left out to other studies.

In order to model the interest rate and asset price channels, the key ingredients that need to be added to the standard production networks model are capital investments by firms and the interest rate. Indeed, both channels rely heavily on their impact on investment, but one of the main assumptions of standard production network models is that all input are used immediately, with no way of accumulating capital. Adding the interest rate is not straightforward since money supply and the interest rate are essentially the same monetary instrument, but the channel may still be modelled by using a capital efficiency function. Ownership of industries by household is also useful to account for wealth effects and is already a part of many production networks models. Note that household dynamics are kept at a minimum in order to focus on firms dynamics. As such, leisure is not modelled and the household has a simple logarithmic utility function focused on consumption, following Carvalho and Tahbaz-Salehi (2019).

These elements are added to the model as follows. Ownership of industries by household can be achieved by adding profits to the household budget constraint (thus adding the wealth effects channel). Capital investments for firms can be modelled by adding capital as a factor of production and defining a capital accumulation function based on the use of intermediate inputs, as theorised by Foerster et al. (2011)[3]. An original way of adding the monetary transmission channels to their investment model is to create a "capital efficiency" function $q(\pi_{it}, M_{it})$ that represents how efficiently firms can transform intermediate inputs into capital. This function is increasing in firms profits (thus adding the Tobin's Q channel) and increasing in money supply (thus adding the interest rate channel, even if the interest rate is not formally modelled). It is taken as given by the firms.

2.2 Model definition

The baseline model used in this analysis is the production network model of Acemoglu et al. (2012), based on the multisector business cycle model of Long and Plosser (1983). The notation and logarithmic utility function follow Carvalho and Tahbaz-Salehi (2019). Capital accumulation is added to the baseline model similarly to Foerster et al. (2011) but with a new capital efficiency parameter q_{it}. Firm ownership by household and some of the dynamic elements are inspired by the Ghassibe (2021b) model.

Multiple sectors/industries compete to produce distinct goods that are either consumed by a representative household, used as intermediate inputs by other industries ("materials"), or used as capital investments by other industries ("capital").

On the production side, $i = 1, ..., n$ industries are competing. They produce an output y_{it} at time t using the Cobb-Douglas production function given in equation 1. Each industry uses labour (l_{it}), capital (k_{it}), and intermediate inputs from other industries (x_{ijt}) as factors of production. z_{it} is a Hicks-neutral supply-side productivity shock. ζ_i is a normalisation constant that simplifies some analytical results in section 3, and is set at $\zeta_i := \alpha_i^{-\alpha_i} \gamma_i^{-\gamma_i} \prod_j \left(a_{ij}^{-a_{ij}} \theta_{ij}^{-\gamma_i \theta_{ij}} \right)$. Production exhibits constant returns to scale so that the factor shares $(\alpha_i, \gamma_i, a_{ij})$ sum up to one: $\alpha_i + \gamma_i + \sum_{j=1}^n a_{ij} = 1$. The shares are assumed to be constant over time.

$$y_{it} = z_{it} \zeta_i l_{it}^{\alpha_i} k_{it}^{\gamma_i} \prod_{j=1}^{n} x_{ijt}^{a_{ij}} \tag{1}$$

Capital evolves according to a standard law of motion (equation 2) with a constant depreciation rate of capital δ. Investments v_{it} are produced using intermediate inputs m_{ijt} with the technology presented in equation 3. The initial value of capital, k_{i0}, is given for all industries. Investments exhibit constant returns to scale so that the input shares (θ_{ij}) sum up to one: $\sum_{j=1}^n \theta_{ij} = 1$. They are also assumed to be constant over time. An additional capital efficiency function is given by a function $q(\pi_{it}, M_t)$, which is increasing in firms profits (π_{it}) and in

[3] Atalay (2017) uses a similar technique with a more complex formulation. The simpler version of Foerster et al. (2011) was used in this paper to focus on the basic dynamics of capital accumulation and restrict the complexity of the model. The main other capital accumulation paper by Vom Lehn and Winberry (2022) uses capital investments by households, which would not fit the monetary transmission channels as well as a model of firms investing intermediate inputs.

money supply (M_t, introduced in section 3). This function parametrises how efficiently industries can transform intermediate inputs into capital. We do not make any assumption on the functional form of the function, apart from it being non-negative. Industries take this function as a given parameter q_{it}.

$$k_{i,t+1} = (1-\delta)k_{it} + q_{it}v_{it} \tag{2}$$

$$v_{it} = \prod_{j=1}^{n} m_{ijt}^{\theta_{ij}} \tag{3}$$

On the household side, a representative agent is endowed with one unit of labour each period which is rented to the industries, so that $\sum_{i=1}^{n} l_{it} = 1$. The household consumes the industries outputs (c_{it}) and has logarithmic preferences over its consumption (equation 4). The share of each good in the household utility function is represented by η_i such that $\sum_{i=1}^{n} \eta_i = 1$.

$$u(c_{1t},...,c_{nt}) = \sum_{i=1}^{n} \eta_i \log\left(\frac{c_{it}}{\eta_i}\right) \tag{4}$$

The household owns all industries. The budget constraint (equation 5) reflects how the price of goods consumed (p_{it}) must be lower than the revenue of the household. The agent receives a wage from its labour endowment (w_t) and receives all the industries profits.

$$\sum_{i=1}^{n} p_{it}c_{it} \leq w_t + \sum_{i=1}^{n} \pi_{it} \tag{5}$$

The industry profits (equation 6) are then determined by the sale of outputs minus labour costs and costs of inputs (materials and capital).

$$\pi_{it} = p_{it}y_{it} - w_t l_{it} - \sum_{j=1}^{n} p_{jt}(x_{ijt} + m_{ijt}) \tag{6}$$

The market clearing condition (equation 7) reflects how goods can be used for consumption, materials, or capital. Note that x_{jit} and m_{jit} describe the output of industry i used as inputs in other industries, while the x_{ijt} and m_{ijt} from equations 1 and 3 describe the output from other industries used as inputs in industry i. As a result, any x_{ijt} or m_{ijt} will enter the social planner problem twice: once as the output of industry j, and once as the input of industry i.

$$y_{it} = c_{it} + \sum_{j=1}^{n} x_{jit} + \sum_{j=1}^{n} m_{jit} \tag{7}$$

2.3 Competitive equilibrium

The addition of the intertemporal capital investment decision for the industries makes the problem dynamic in nature and hard to solve analytically, since industries need to maximise their intertemporal profits instead of having a static problem. While it is tempting to use a recursive competitive equilibrium approach to simplify the capital accumulation process, the network structure of the model adds a lot of complexity to dynamic programming. A standard approach for production networks is instead to solve for the social planner equilibrium since its optimal allocation is equivalent to the competitive equilibrium by the first welfare theorem (Negishi, 1960). As such, an Arrow-Debreu time 0 approach is used in this paper, with industries optimising their total lifetime profits.

Arrow-Debreu competitive equilibrium

An Arrow-Debreu competitive equilibrium in this investment model is a set of sequences $\{(c_{it}, l_{it}, k_{i,t+1}, p_{it}, \pi_{it}, q_{it})_{i=1,...,n}, (x_{ijt}, m_{ijt})_{i,j=1,...,n}, w_t\}_{t=0}^{\infty}$ such that the following hold.

Taking $\{w_t, (p_{it}, \pi_{it})_{i=1,...,n}\}_{t=0}^{\infty}$ as given, $\{(c_{it})_{i=1,...,n}\}_{t=0}^{\infty}$ solves the representative household lifetime utility maximisation problem:

$$\begin{aligned}
\max_{\{(c_{it})_{i=1,...,n}\}_{t=0}^{\infty}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \left[\beta^t \sum_{i=1}^{n} \eta_i \log\left(\frac{c_{it}}{\eta_i}\right) \right] \\
\text{s.t.} \quad & \sum_{t=0}^{\infty} \left[\sum_{i=1}^{n} p_{it} c_{it} \right] \leq \sum_{t=0}^{\infty} \left[w_t + \sum_{i=1}^{n} \pi_{it} \right] \\
& c_{it} \geq 0, \qquad\qquad \forall i, t
\end{aligned}$$

Taking $\{w_t, k_{i0}, q_{it}, (p_{jt})_{j=1,...,n}\}_{t=0}^{\infty}$ as given, $\{l_{it}, k_{i,t+1}, (x_{ijt}, m_{ijt})_{j=1,...,n}\}_{t=0}^{\infty}$ solves the profit maximisation problem of each industry $i = 1, .., n$:

$$\begin{aligned}
\pi_i^{tot} = \max_{\{l_{it}, k_{i,t+1}, (x_{ijt}, m_{ijt})_{j=1,...,n}\}_{t=0}^{\infty}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \left[p_{it} y_{it} - w_t l_{it} - \sum_{j=1}^{n} p_{jt} (x_{ijt} + m_{ijt}) \right] \\
\text{s.t.} \quad & y_{it} = z_{it} \zeta_i l_{it}^{\alpha_i} k_{it}^{\gamma_i} \prod_{j=1}^{n} x_{ijt}^{a_{ij}}, \qquad \forall t \\
& k_{i,t+1} = (1-\delta) k_{it} + q_{it} v_{it}, \qquad \forall t \\
& v_{it} = \prod_{j=1}^{n} m_{ijt}^{\theta_{ij}}, \qquad \forall t \\
& 0 \leq l_{it} \leq 1, x_{ijt} \geq 0, m_{ijt} \geq 0, \qquad \forall i, t
\end{aligned}$$

The market clearing conditions for the goods markets and labour market hold:

$$y_{it} = c_{it} + \sum_{j=1}^{n} x_{jit} + \sum_{j=1}^{n} m_{jit}, \quad \forall i, t$$

$$\sum_{i=1}^{n} l_{it} = 1$$

Social planner equilibrium

$\left\{(c_{it}, l_{it}, k_{i,t+1})_{i=1,\dots,n}(x_{ijt}, m_{ijt})_{i,j=1,\dots,n}\right\}_{t=0}^{\infty}$ solves the social planner problem:

$$\begin{aligned}
\max_{\left\{(c_{it},l_{it},k_{it})_{i=1,\dots,n},(x_{ijt},m_{ijt})_{i,j=1,\dots,n}\right\}_{t=0}^{\infty}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty}\left[\beta^t \sum_{i=1}^{n} \eta_i \log\left(\frac{c_{it}}{\eta_i}\right)\right] \\
\text{s.t.} \quad & y_{it} = c_{it} + \sum_{j=1}^{n} x_{jit} + \sum_{j=1}^{n} m_{jit}, && \forall i,t \\
& y_{it} = z_{it}\zeta_i l_{it}^{\alpha_i} k_{it}^{\gamma_i} \prod_{j=1}^{n} x_{ijt}^{a_{ij}}, && \forall i,t \\
& k_{i,t+1} = (1-\delta)k_{it} + q_{it}\prod_{j=1}^{n} m_{ijt}^{\theta_{ij}}, && \forall i,t \\
& 0 \le l_{it} \le 1, c_{it} \ge 0, x_{ijt} \ge 0, m_{ijt} \ge 0, && \forall i,t \\
& \sum_{i=1}^{n} l_{it} = 1, && \forall t
\end{aligned}$$

Equilibrium allocations

The Lagrangian function is provided in appendix A. The λ_{it} are the Lagrange multipliers for the resource constraints, the μ_{it} are the Lagrange multipliers for the capital accumulation constraints, and the ϕ_t are the Lagrange multipliers for the labour supply constraints. The first order conditions of the social planner problem are:

$$\begin{aligned}
c_{it}: \quad & \lambda_{it} = \beta^t \eta_i \frac{1}{c_{it}} \\
l_{it}: \quad & \phi_t = \lambda_{it}\alpha_i \frac{y_{it}}{l_{it}} \\
k_{i,t+1}: \quad & \mu_{it} = \lambda_{i,t+1}\gamma_i \frac{y_{i,t+1}}{k_{i,t+1}} + (1-\delta)\mu_{i,t+1} \\
x_{ijt}: \quad & x_{ijt} = \frac{\lambda_{it}}{\lambda_{jt}} a_{ij} y_{it} \\
m_{ijt}: \quad & \lambda_{jt} = \mu_{it}\theta_{ij} \frac{q_{it} v_{it}}{m_{ijt}}
\end{aligned}$$

Where the endogenous variables are $\{x_{ijt}, m_{ijt}, c_{it}, l_{it}, y_{it}, k_{i,t+1}, v_{it}, \pi_{it}, \lambda_{it}, \mu_{it}, \phi_t\}$, for a total size of $2n^2+8n+1$ unknowns each period. As such, the $2n^2+3n$ first order conditions, together with the $n+1$ market clearing conditions (eq. 7 and $\sum l_{it} = 1$), n production functions (eq. 1), n firm profit functions (eq. 6), n laws of motion of capital (eq. 2), and n investment functions (eq. 3) are sufficient to solve the system.

In addition to the social planner approach, the relative prices of the model can be obtained by taking the first order condition of the profit maximisation problem. The conditions for x_{ijt} and l_{it} are straightforward. To derive m_{ijt}, we need to account for the fact that intermediate capital inputs enter the profit maximisation problem twice: once in time t as a cost, and once in time

$t+1$ as a factor of production. The same goes for the $k_{i,t+1}$ condition, which is more complex to derive as we need to account for the capital accumulation condition. The best way to do so is to derive the shadow value of the capital accumulation constraint using a Lagrangian and use it to solve its first order condition. This derivation is shown in appendix B. Note that the optimisation approach follows Foerster et al. (2011), where firms optimise next period capital $k_{i,t+1}$ and then chose the optimal intermediate inputs m_{ijt} to achieve it. The industries first order conditions are then:

$$x_{ijt} = \frac{p_{it}}{p_{jt}} a_{ij} y_{it} \tag{8}$$

$$l_{it} = \frac{p_{it}}{w_t} \alpha_i y_{it} \tag{9}$$

$$m_{ijt} = \frac{p_{i,t+1}}{p_{jt}} \gamma_i \theta_{ij} y_{i,t+1} \frac{q_{it} v_{it}}{k_{i,t+1}} \tag{10}$$

$$k_{i,t+1} = p_{i,t+1} \gamma_i y_{i,t+1} \frac{q_{it} q_{i,t+1} \prod_j \theta_{ij}^{\theta_{ij}}}{(1-\delta) q_{i,t+1} \prod_j p_{jt}^{\theta_{ij}} - q_{it} \prod_j p_{j,t+1}^{\theta_{ij}}} \tag{11}$$

The intuition for these conditions is as follows. Firms make their optimal input consumption choice for every potential trade partner j (including themselves as self-consumption) by choosing x_{ijt} based on how relatively expensive the input is, how efficient it is in production with the materials linkage a_{ij}, and the optimal production value. Similarly, they chose their optimal labour input l_{it} based on how relatively expensive wages are, how efficient labour is in production with the share α_i, and the optimal production value. The choice of capital includes dynamic elements since firms make their $t+1$ capital decision in t by buying the required inputs. Its optimal level depends on the relative prices of inputs, capital efficiency q_{it}, the share of capital in production γ_i, capital depreciation δ, and the capital linkages θ_{ij}. The optimal intermediate input used for capital investment is then a function of the same parameters, with the relative prices of inputs being weighted based on their intertemporal efficiency.

2.4 Industry linkages and network structure

The industry linkages can be summarised by two $n \times n$ matrices. First, the input-output matrix $A = [a_{ij}]$ which describes the importance of the outputs as intermediate goods for other industries (materials m). Second, the investment matrix $\Theta = [\theta_{ij}]$ which describes the importance of the outputs for the capital investments of other industries (capital k). These matrices can be redefined by their respective Leontief inverse matrix L^m, L^k, where each typical element ℓ_{ij}^m, ℓ_{ij}^k measures the importance of j as a direct and indirect input supplier of i:

$$L^m := (I_n - A)^{-1}$$
$$L^k := (I_n - \Theta)^{-1}$$

The Leontief inverses are a measure of sectoral importance because they account for the role of industry i as a direct supplier of j, as a supplier of suppliers of j, as a supplier of suppliers of suppliers of j, and so on (Carvalho and Tahbaz-Salehi, 2019):

$$\ell_{ij}^{m} = a_{ij} + \sum_{k=1}^{n} a_{ik} a_{kj} + \sum_{k=1}^{n} \sum_{r=1}^{n} a_{ik} a_{kr} a_{rj} + \ldots$$
$$\ell_{ij}^{k} = \theta_{ij} + \sum_{k=1}^{n} \theta_{ik} \theta_{kj} + \sum_{k=1}^{n} \sum_{r=1}^{n} \theta_{ik} \theta_{kr} \theta_{rj} + \ldots$$

Both linkages matrices can be represented as networks, namely the production network and the investment network. This results in two weighted directed graphs, each with n vertices (industries) and a maximum of $n(n-1)$ directed edges with weights a_{ij} or θ_{ij} (the input-output linkages). As such, the Leontief inverse measures the product of the weights of each possible path with industry i as the last vertex.

3 Monetary transmission

This section analyses how to add money supply to the model defined in section 2, the resulting propagation patterns, and a simplified three-sectors example to illustrate the propagation mechanisms.

3.1 Money supply shock

A standard way to add money supply to a business cycle model is to use the cash-in-advance assumption of Clower (1967). This approach is also used by Ozdagli and Weber (2017) and Di Giovanni and Hale (2022). The assumption is based on the idea that the household buys the consumption goods with cash, while the industries use trade credit to purchase inputs. As a result, money supply (M_t) can be incorporate into the model as the sum of household purchases:

$$M_t \equiv \sum_{i=1}^{n} p_{it} c_{it}$$

In the baseline production network model, shocks can either propagate upstream (from customers to suppliers) or downstream (from suppliers to customers). This is a result of the directed nature of the network. For instance, an household demand shock will increase the production of the final goods producers. These industries will ask for more factors of production and increase their demand of intermediate goods. As a result, their suppliers increase their intermediate demand to increase production, the suppliers of their suppliers increase their intermediate demand, and so on, creating an upstream cascade of shocks. The downstream effect works in the same cascade spirit but with customers instead of suppliers. The literature, such as Acemoglu et al. (2016), makes a clear distinction between demand-side shocks that exclusively propagate upstream and supply-side shocks that exclusively propagate downstream. Moreover, Ozdagli and Weber (2017) study exclusively the upstream propagation of monetary shocks.

One of the main consequences of modifying the model to include capital accumulation and capital efficiency is that it allows for money supply shocks to propagate both upstream and downstream. This new approach allows for more complex propagation patterns to emerge, and for a more detailed empirical analysis.

First, money supply shocks propagate upstream because of the wealth effect channel and the cash-in-advance assumption (theorem 1). This makes money supply shocks close to the demand-side shocks of Acemoglu et al. (2016), with the addition of the role of investments linkages in the industries output. Input-output linkages have a large role to play in the propagation of shocks as the consumption shock affects the output of the household suppliers, which then affects their suppliers, and so on. In the model, this phenomenon is captured by how the total industry output depends on money supply and capital efficiency via both the materials and capital linkages:

Theorem 1 (Upstream propagation). *Let L^m be the Leontief inverse of the material linkages and Θ be the the capital linkages. The total output is:*

$$\sum_{i=1}^{n} p_{it} y_{it} = M_t + \sum_{i=1}^{n}\sum_{j=1}^{n} p_{jt} a_{ji} y_{jt} + \sum_{i=1}^{n}\sum_{j=1}^{n} p_{j,t+1} \gamma_j \theta_{ji} y_{j,t+1} \frac{q(\pi_{jt}, M_t) v_{jt}}{k_{j,t+1}}$$

And the output of all industries is:

$$\begin{bmatrix} p_{1t} y_{1t} \\ \vdots \\ p_{nt} y_{nt} \end{bmatrix} = L^{m'} \begin{bmatrix} p_{1t} c_{1t} \\ \vdots \\ p_{nt} c_{nt} \end{bmatrix} + L^{m'} \Theta' \left(\begin{bmatrix} p_{1,t+1} y_{1,t+1} \\ \vdots \\ p_{n,t+1} y_{n,t+1} \end{bmatrix} \circ \begin{bmatrix} \gamma_1 q(\pi_{1t}, M_t) v_{1t} / k_{1,t+1} \\ \vdots \\ \gamma_n q(\pi_{nt}, M_t) v_{nt} / k_{n,t+1} \end{bmatrix} \right)$$

Hence, money supply shocks propagate upstream.

Proof. See appendix C

Second, money supply shocks propagate downstream because of the interest rate channel and Tobin's Q channels via the q_{it} function (theorem 2). This makes money supply shocks close to the supply-side shocks of Acemoglu et al. (2016), with the addition of capital linkages in the determination of relative prices. By reducing the cost of capital, the shocks increase relative prices in a similar way as a technology shock z_{it}. Input-output linkages are once again important for the propagation of monetary shocks, as the technology shock affect the industries clients, which affect their own clients, and so on. In the model, this phenomenon is captured by how the money supply shocks are able to change relative prices via their effect on capital efficiency, multiplied by the Leontief inverse of both the materials and capital linkages:

Theorem 2 (Downstream propagation). *Let A, Θ be the material linkages and capital linkages. Relative prices are determined by the supply-side technology (z_{it}) and money supply via capital efficiency (q_{it}):*

$$\begin{bmatrix} \log(p_{1t}/w_t) \\ \vdots \\ \log(p_{nt}/w_t) \end{bmatrix} = -(I_n - A - \Theta)^{-1} \left(\begin{bmatrix} \log(z_1) \\ \vdots \\ \log(z_n) \end{bmatrix} + \begin{bmatrix} \gamma_1 \log(q_{1t} q_{1,t-1}) \\ \vdots \\ \gamma_n \log(q_{nt} q_{n,t-1}) \end{bmatrix} \right)$$

Hence, money supply shocks propagate downstream.

Proof. See appendix D

Adding investments to the model changes relative prices. In the baseline model, prices are determined by the supply-side shocks exclusively[4]: $P_t = -(I_n - A)^{-1} \epsilon_t$ (Carvalho and Tahbaz-Salehi, 2019). But in the modified model, relative prices also depend on the efficiency of capital[5]: $P_t = -(I_n - A - \Theta)^{-1}(\epsilon_t + \psi_t)$ (theorem 2). As a result, we must not only consider input-output linkages in the trade of materials (A) but also in the trade of capital (Θ) to not underestimate the role of input-output linkages in the propagation of shocks.

3.2 Three-sectors example

It is useful to consider a simplified example in order to illustrate the propagation of monetary shocks in the modified model. Let the economy be composed of three industries ($i = 1, 2, 3$) and a representative household. Industry 1 is a supplier of materials for both 2 and 3. Industry 1 supplies capital to 2, industry 2 supplies capital to 3, and industry 3 supplies capital to 1. The household consumes the good of industries 2 and 3. This example is shown in figure 2.

In this model, the output and capital of the industries are given by:

$$\begin{aligned} y_{1t} &= z_{1t}\zeta_1 l_{1t}^{\alpha_1} k_{1t}^{\gamma_1}, & k_{1,t+1} &= (1-\delta)k_{1t} + q_{1t} m_{13t}^{\theta_{13}} \\ y_{2t} &= z_{2t}\zeta_2 l_{2t}^{\alpha_2} k_{2t}^{\gamma_2} x_{21t}^{a_{21}}, & k_{2,t+1} &= (1-\delta)k_{2t} + q_{2t} m_{21t}^{\theta_{21}} \\ y_{3t} &= z_{3t}\zeta_3 l_{3t}^{\alpha_3} k_{3t}^{\gamma_3} x_{31t}^{a_{31}}, & k_{3,t+1} &= (1-\delta)k_{3t} + q_{3t} m_{32t}^{\theta_{32}} \end{aligned}$$

And the market clearing conditions are:

$$\begin{aligned} y_{1t} &= x_{21t} + x_{31t} + m_{21t} \\ y_{2t} &= c_{2t} + m_{32t} \\ y_{3t} &= c_{3t} + m_{13t} \end{aligned}$$

It is immediate from the market clearing conditions that a consumption shock caused by money supply will impact the output of industries 2 and 3 directly, as they are the suppliers of the household. Upstream propagation happens if the output of industry 3, the supplier of the sup-

[4]Where $P_t = [\log(p_{1t}/w_t), ..., \log(p_{nt})/w_t]$ and $\epsilon_t = [\log(z_1), ..., \log(z_n)]$
[5]Where $\psi_t = [\gamma_1 \log(q_{1t} q_{1,t-1}), ..., \gamma_n \log(1_{nt} q_{n,t-1})]$

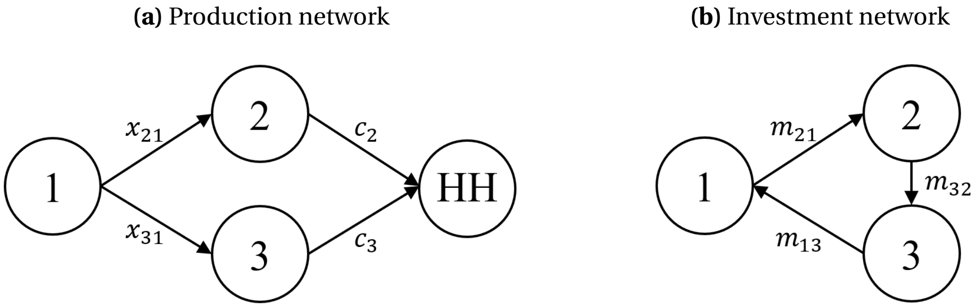

Figure 2: Theoretical example of a network with 3 sectors and a representative household (HH)

pliers, is also affected by the shock. Taking the firms first order conditions (eq. 8, 10) and substituting them in the market clearing conditions, we can see that this is indeed the case:

$$y_{1t} = \frac{p_{2t}}{p_{1t}} a_{21} y_{2t} + \frac{p_{3t}}{p_{1t}} a_{31} y_{3t} + \left(\frac{p_{2,t+1}}{p_{1t}} \theta_{21} \gamma_2 q_{2t} \frac{y_{2,t+1}}{k_{2,t+1}} \right)^{\frac{1}{1-\theta_{21}}}$$

Where the first two terms show how the consumption shock propagates through the input-output linkages in materials (a_{ij}). The third term shows how industry 1 being the capital supplier of industry 2 yields an additional level of interconnectedness in the model, with the shock propagating through the input-output linkages in capital (θ_{ij}). Overall, the money supply shock on the household has a positive effect on its suppliers, sectors 2 and 3, which then have a positive effect on their supplier, sector 1. This shows how money supply shocks propagate upstream.

Downstream propagation is also present in this example. A money supply shock will impact all industries, but let us focus on its impact on industry 1. It is immediate from the first order condition for capital that the shocks increases the next-period capital for industry 1 since it is able to generate more capital from the same intermediate input:

$$k_{1,t+1} = p_{1,t+1} \gamma_1 y_{1,t+1} \frac{q(\pi_{1t}, M_t) q(\pi_{1,t+1}, M_{t+1}) \theta_{13}^{\theta_{13}}}{(1-\delta) q(\pi_{1,t+1}, M_{t+1}) p_{3t}^{\theta_{13}} - q(\pi_{1t}, M_t) p_{3,t+1}^{\theta_{13}}}$$

Downstream propagation happens if the shock propagates to its customers: industries 2 and 3. This is indeed the case since capital is a factor of production, so that the higher investments lead to a higher production y_{1t} by the production function of industry 1. All things equal, this makes the relative price of good 1 lower since it is more abundant in the economy. As a result, sectors 2 and 3 will buy more intermediate goods from industry 1 due to their lower price, as per their first order conditions (eq. 8, 10):

$$\begin{aligned} x_{21t} &= \frac{p_{2t}}{p_{1t}} a_{21} y_{2t} \\ x_{31t} &= \frac{p_{3t}}{p_{1t}} a_{31} y_{3t} \\ m_{21t} &= \frac{p_{2,t+1}}{p_{1t}} \gamma_2 \theta_{21} y_{2,t+1} \frac{q_{2t} v_{2t}}{k_{2,t+1}} \end{aligned}$$

In a model with more sectors, industries 2 and 3 would then propagate the shocks to their customers in a similar manner since the higher inputs increase their own productions. It is immediate from the household budget constraint that the change of relative prices will also increase its utility and consumption decision. Once again, the shock propagate through the input-output linkages in materials (a_{ij}), and the fact that industry 2 buys capital from industry 1 yields an additional level of interconnectedness through the input-output linkages in capital (θ_{ij}). As a result, the money supply shock on sector 1 has a positive effect on its customers, sectors 2 and 3, and the household. This shows how money supply shocks propagate downstream.

4 Empirical analysis

This section presents an empirical analysis quantifying the role of network effects in the transmission of money supply shocks, using autoregressions on the 2005-2018 US input-output data.

4.1 Data

In order to match the model, the empirical analysis needs data on material linkages (A), capital linkages (Θ), sectoral production (y_{it}), and money supply (M_t).

Data on the investment network are hard to come by. While the US Bureau of Economic Analysis (BEA) provides sectoral capital flows tables that would fit this definition, the data is sparse, inconsistent, and does not include all intellectual property. The alternative proposed by Vom Lehn and Winberry (2022) is to compute an investment network based on the BEA asset-level data. As such, they provide a yearly estimate of capital flows between 37 manufacturing[6] and services[7] sectors in the US for the 1947-2018 period. This paper uses their data as capital linkages.

Data on the production network is directly available from the BEA input-output tables and is commonly used in the macro networks literature. In order to match the investment network, the material linkages focus on the same 37 sectors in the US for the 1947-2018 period, even if a more granular sector decomposition is available (as used in figure 1).

Data on sectoral production is obtained from the BEA GDP-by-industry tables. They provide the seasonally adjusted gross output for each industry quarterly for the Q1.2005-Q1.2022 period. While some datasets are available before this period, they can only be found yearly, which would not suit the frequency of money supply shocks.

Lastly, data on money supply is taken from the Federal Reserve Bank of St. Louis MZM Money

[6] Manufacturing sectors: mining, utilities, construction, wood products, nonmetallic minerals, primary metals, fabricated metals, machinery, computer and electronic, electrical equipment, transportation equipment, motor vehicles, miscellaneous manufacturing, furniture, textile, food and beverages, paper, apparel, printing products, petroleum and coal, chemical, plastics

[7] Services sectors: wholesale trade, retail trade, transportation, information, finance and insurance, real estate and rental, professional and technical services, management, administrative and waste management, education, health care and social assistance, arts and entertainment, accommodation, food services, miscellaneous services

(a) Production network

(b) Investment network

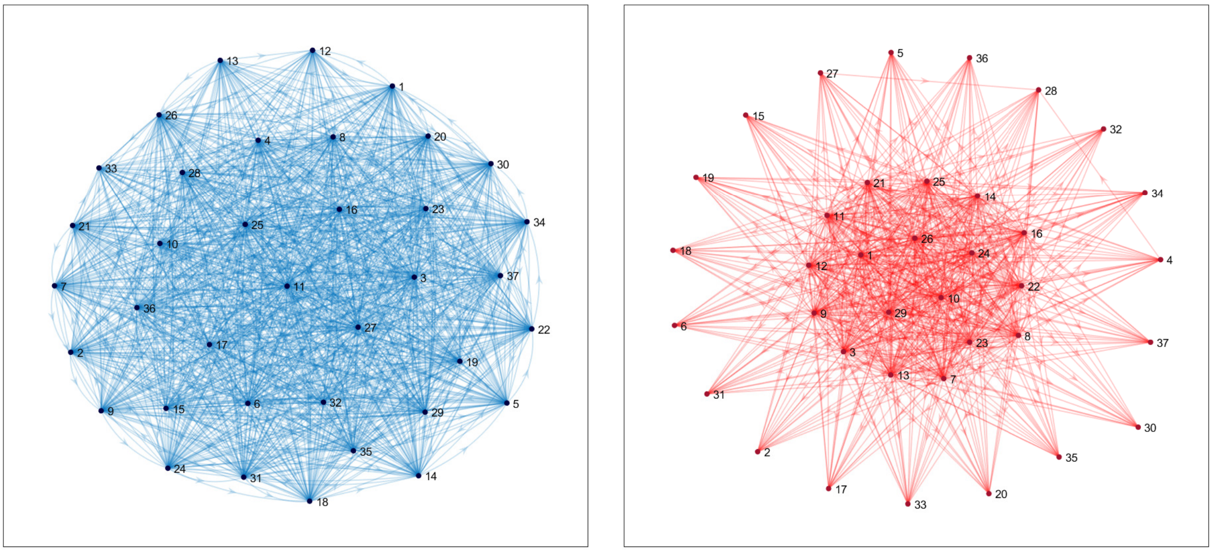

Figure 3: US production and investment networks of 37 sectors in 2018. *Notes:* Directed weighted graphs generated from the BEA input-output tables and investment network data of Vom Lehn and Winberry (2022). Each vertex represents a sector and each edge represents a directed trade relation or capital flow with another sector. Self-loops are omitted.

Stock series. While there are several narrow and broad definitions of money supply, money zero maturity (MZM) is commonly used in the monetary economics literature as it measures all liquid money supply. The data contain monthly observations for the Jan.1959-Jan.2021 period. Since only quarterly observations are needed, the end-of-period money supply of each quarter is used. The money supply shocks ($\widehat{M}_t$) are the quarterly change of money supply with respect to the previous period.

The data needs to be modified to perform the regressions. First, one sector needs to be dropped to avoid collinearity issues since the sum of input-output linkages is fixed to 1 for all sectors. The *miscellaneous services* was chosen for that since it is the smallest of the two miscellaneous categories, with little trade to other sectors. Second, the investment network data contain multiple rows and columns of zeroes since some sectors do not produce capital. To avoid a rank deficient matrix, the zeroes are replaced with small random numbers drawn from a normal distribution $N(10^{-5}, 10^{-5})$, which matches what Foerster et al. (2011) did to solve the same issue. Both of these modifications are rather small and should not have a significant impact on the results.

Overall, this gives a quarterly dataset of the production of 36 sectors and the money supply shocks for the Q1.2005-Q4.2018 period, as well as a yearly dataset of intersectoral linkages in materials and capital for the 1947-2018 period. The two empirical networks in 2018 are shown in figure 3, which indicates how the production network has a near-complete structure while the investment network has a core-periphery structure. This highlights how interconnectedness may be more important for materials, while capital production is concentrated in a smaller hub of sectors.

4.2 Empirical strategy

Before estimating the network effects, the model assumption that the input-output matrices are constant over time needs to be empirically verified. This can be checked by fitting simple autoregressions on the $2n^2 = 2738$ input-output linkages:

$$a_{ijt} = c^a_{ij} + \rho^a_{ij} a_{ij,t-1} + \epsilon^a_{ijt}$$
$$\theta_{ijt} = c^\theta_{ij} + \rho^\theta_{ij} \theta_{ij,t-1} + \epsilon^\theta_{ijt}$$

Which are constant up to random errors if $\rho_{ij} = 1$. This can be tested with an augmented Dickey-Fuller test for a unit root, and the results are presented in table 1. Taking the full 1947-2018 sample, the unit root null $\rho_{ij} = 1$ cannot be rejected at $\alpha = 10\%$ for 75.6% of the materials coefficients and for 91.1% of capital coefficients. As such, there are some long-term changes in approximately one-fourth of the input-output linkages of the US economy, even though capital linkages seem to be more stable in the long run than materials linkages. However, taking only the 2005-2018 period that is used in this paper, the null cannot be rejected for 91.9% of the materials coefficients and for 89.7% of capital coefficients. This shows that the input-output linkages are rather stable in the short term for both materials and capital, and that the assumption of constant coefficients holds well for this period. The analysis will thus use a fixed input-output matrix as the reference linkages A, Θ.

Table 1: Test of constant input-output linkages

Null of unit root	Materials linkages		Capital linkages	
	1947-2018	2005-2018	1947-2018	2005-2018
Cannot reject at $\alpha = 10\%$	1034	1258	1247	1228
Reject at $\alpha = 10\%$	335	111	122	141
Reject at $\alpha = 5\%$	263	69	73	101
Reject at $\alpha = 1\%$	179	27	20	56

Notes: Individual augmented Dickey-Fuller (ADF) test of unit roots performed on all 2×1369 coefficients a_{ijt}, θ_{ijt}. The null hypothesis is the presence of a unit root. The tests are performed using US yearly input-output data for 1949-2018 or the 2005-2018 subsample. The table reports the number of coefficients with an ADF p-value (i) above 0.1, (ii) below 0.1, (iii) below 0.05, (iv) below 0.01.

The main goal of the empirical analysis is to estimate the share of network effects in the total transmission of money supply shocks to production. This is close to the goal of Di Giovanni and Hale (2022) and Ozdagli and Weber (2017) which estimate network effects in the transmission of money supply shocks to asset prices, so that a similar strategy can be used. The effect can be estimated using spatial autoregressions that regress gross sectoral output on money supply shocks. Then, the estimate can be decomposed into a direct and indirect effect using the Acemoglu et al. (2016) decomposition. Finally, the network share is the share of indirect effects in the total effect:

$$\text{Network share}_i = \frac{\text{Indirect effect}_i}{\text{Direct effect}_i + \text{Indirect effect}_i}$$

The first regression is based on the production network alone and ignores the effect of the investment network. This provides some baseline results to understand how important materials linkages are, and it allows for comparisons with other studies that do not use the investment network. The regression[8] is presented in equation 12 and is a simple adaptation of Di Giovanni and Hale (2022) but with gross output instead of asset prices. A constant α_i is used for each sector to capture fixed effects. Following the Acemoglu et al. (2016) decomposition, the direct effect is β, the total effect is $L^m\beta$, and the indirect effect is the difference between the two.

$$Y_t = \alpha + L^m \widehat{M}_t \beta + \epsilon_t \tag{12}$$

The second regression accounts for the effect of both the production and investment networks. The previous autoregression should thus be modified to account for the fact that money supply shocks propagate through both materials and capital linkages. A simple way to do this is to add the capital Leontief inverse to the material Leontief inverse. As each matrix represents the importance of sectoral linkages in each network, the sum of the two becomes the total measure of linkages including the investment network. The regression[9] is presented in equation 13 and also uses fixed effects. Adapting the Acemoglu et al. (2016) decomposition for the investment network, the direct effect is β, the total effect is $(L^m + L^k)\beta$, and the indirect effect is the difference between the two.

$$Y_t = \alpha + \left(L^m + L^k\right) \widehat{M}_t \beta + \epsilon_t \tag{13}$$

Both autoregressions compute the network share of the transmission of money supply shocks to production for each sector of the economy. Following Di Giovanni and Hale (2022) and Ozdagli and Weber (2017), the results are then summarised by taking the average network share over all sectors.

Lastly, one can see which of the downstream or upstream effect dominates using the Acemoglu et al. (2016) empirical approach. This third regression is presented in equation 14 and separates the Leontief inverses between downstream and upstream coefficients. That way, the ratio of the coefficients β_{up} and β_{down} allows to understand which propagation direction is most important in the case of monetary shocks. All of the coefficients $\left(\omega, \psi, \beta_{up}, \beta_{down}\right)$ are scalars in this new setup.

$$Y_{it} = \omega + \psi Y_{i,t-1} + \beta_{up}\text{Upstream}_{it} + \beta_{down}\text{Downstream}_{it} + \epsilon_t \tag{14}$$

$$\text{Upstream}_{it} \equiv \sum_{j=1}^{n} \left[(\ell^m_{i\to j} + \ell^k_{i\to j}) - \mathbb{1}_{i=j} \right] \widehat{M}_t$$

$$\text{Downstream}_{it} \equiv \sum_{j=1}^{n} \left[(\ell^m_{j\to i} + \ell^k_{j\to i}) - \mathbb{1}_{i=j} \right] \widehat{M}_t$$

[8] Where $Y_t = \left[y_{1t},...,y_{nt}\right]'$, $\alpha = [\alpha_1,...,\alpha_n]'$, $L^m = \left\{\ell^m_{ij}\right\}$, $\widehat{M}_t$ a scalar, $\beta = \left[\beta_1,...,\beta_n\right]'$, $\epsilon_t = [\epsilon_{1t},...,\epsilon_{nt}]'$

[9] Where $L^k = \left\{\ell^k_{ij}\right\}$

4.3 Results

Table 2 reports the results of regressions 12 and 13 decomposed by sector, using 2018 as the reference year for input-output linkages. It shows how the importance of network effects varies significantly across sectors, indicating an important heterogeneity in the role of input-output linkages in the transmission of monetary shocks. These estimates are consistent with the results of Di Giovanni and Hale (2022), which also found large differences between sectors.

Table 3 reports the average network share across all sectors. The regressions are run three times using different reference years for the input-output linkages matrices: using the linkages at the beginning of observations (2005), at the midpoint (2011), and at the end of the observations (2018). It shows that 40-59% of the total effect can be attributed to network effects depending on the reference year and whether we account for capital linkages. It also shows how approximately one-fifth of the network effects appears only in the second regression, so that ignoring the investment network by using only the production network would underestimate the network share. This result is in line with Atalay (2017) which found that ignoring capital linkages underestimates the network amplification of micro shocks. Lastly, it shows how sectoral linkages became more important for the transmission of shocks in recent years, as the network shares increase with the reference year. These estimates are slightly below Ozdagli and Weber (2017) which reports a network share of 50-85%, and Di Giovanni and Hale (2022) which reports a network share of 56% for the US, both without the investment network. This could indicate that network effects are more important in the transmission of money supply shocks to asset prices than they are for the transmission to production. This can be consistent with the monetary transmission channels, as the impact on asset prices is the first step of monetary transmission while the impact on production involves additional channels where some of the effect could be lost, or where some additional unobserved effects could dilute the share of network effects. This explanation is in line with the New-Keynesian model results of Ghassibe (2021b) which found that only 30% of the amplification effects of monetary shocks on aggregate consumption could be attributed to network effects, which is significantly lower than for asset prices.

These empirical results also cast doubt on the assumption of a Cobb-Douglas production function (eq. 1). The sectoral results of table 2 include network shares that are negative and above 100% in absolute value, which is also the case in Di Giovanni and Hale (2022). Estimates above 100% may be surprising but are consistent with a Cobb-Douglas form. They simply indicate that the direct and indirect effects are of opposite signs with the indirect effect dominating. For instance, this can be the case in a sector where the money supply shock has a negative direct impact on its production and a positive impact on most of its suppliers, so that trade makes the total impact positive. However, negative network effects are inconsistent with a Cobb-Douglas production, as they require opposite signs with the direct effect dominating. Indeed, this propagation pattern cannot be generated with a Cobb-Douglas due to its fixed ratio of input expenditures to sales (Carvalho and Tahbaz-Salehi, 2019). This discrepancy can be solved with a constant elasticity of substitution (CES) function or by implementing sticky prices, which

Table 2: Sectoral network effects of money supply shocks

	Production network			Prod. & Invest. networks		
Sector	Direct effect	Indirect effect	Network share	Direct effect	Indirect effect	Network share
Mining	-444929	1039602	1.75	-18070	612742	1.03
Utilities	294206	-67427	-0.30	100031	126748	0.56
Construction	334388	-92437	-0.38	617883	-375933	-1.55
Wood products	-69714	-13493	0.16	-63305	-19902	0.24
Nonmetallic minerals	23375	43618	0.65	-12475	79468	1.19
Primary metals	-9519	474555	1.02	-23151	488186	1.05
Fabricated metals	265209	222929	0.46	93451	394687	0.81
Machinery	404293	139005	0.26	343406	199891	0.37
Electronic	323801	162797	0.33	306899	179699	0.37
Electrical equipment	86720	72693	0.46	22248	137165	0.86
Motor vehicles	-303785	-159196	0.34	-43812	-419169	0.91
Transportation eqpmt.	108040	78531	0.42	83544	103027	0.55
Furniture	50463	1845	0.04	60015	-7707	-0.15
Miscellaneous	130396	-39822	-0.44	127901	-37326	-0.41
Food and beverages	59247	-145105	1.69	-1862	-83996	0.98
Textile	151	-15156	1.01	-10184	-4821	0.32
Apparel	-9627	-9780	0.50	-7856	-11550	0.60
Paper	46493	39839	0.46	15613	70718	0.82
Printing	129588	-29401	-0.29	50822	49365	0.49
Petroleum and coal	994015	134144	0.12	486116	642042	0.57
Chemical	300564	282169	0.48	140676	442056	0.76
Plastics	-29484	5316	-0.22	-55153	30985	-1.28
Wholesale trade	-575869	194040	-0.51	-213636	-168193	0.44
Retail trade	-818807	23395	-0.03	-385250	-410162	0.52
Transportation	168473	93238	0.36	991	260721	1.00
Information	-278716	-308461	0.53	-117236	-469942	0.80
Finance and insurance	87871	-667859	1.15	-173885	-406102	0.70
Real estate	-209666	-781193	0.79	-441220	-549640	0.55
Professional services	358364	-541697	2.95	312799	-496132	2.71
Management	105303	-31201	-0.42	-71414	145516	1.96
Administrative	219846	-398450	2.23	-93747	-84858	0.48
Education	-113635	-22647	0.17	-63383	-72899	0.53
Health care	-1097251	-43378	0.04	-559107	-581521	0.51
Entertainment	-344275	-124899	0.27	-205698	-263476	0.56
Accommodation	-41878	-28001	0.40	-34973	-34906	0.50
Food services	-228570	-113626	0.33	-158481	-183716	0.54

Notes: Regression of sectoral outputs on money supply shocks using regressions 12 (production network only) and 13 (production and investment networks). The decomposition of the network effects follows Acemoglu et al. (2016), where the β are the direct effects, $L^m\beta$ and $\left(L^m + L^k\right)\beta$ are the total effects, and the indirect effect is the difference between the two. The network share is the share of indirect effects over total effects. The reference year for I-O linkages matrices is 2018.

Table 3: Average network effects of money supply shocks

I-O reference year	Production network	Prod. & Invest. networks
2005	0.40	0.53
2011	0.44	0.59
2018	0.47	0.58

Notes: Average sectoral share of indirect effects in the total effect of money supply shocks on output. The regressions follow equations 12 and 13, and the network shares are computed using the Acemoglu et al. (2016) decomposition.

would relax the constant expenditure assumption.

Lastly, table 4 reports the result of regression 14, decomposing the effect in a downstream and upstream parts using the 2018 input-output matrices as references. It is immediate from the results that downstream effects are more important than upstream effects. More specifically, they are almost 2.6 times bigger. This result outlines how important the investment mechanism is for monetary transmission, since it is what drives downstream propagation as per theorem 2. This provides confidence in the modified model that includes capital accumulation and its efficiency via the function $q(\pi_{it}, M_t)$ to account for these effects.

Table 4: Upstream/downstream decomposition of money supply shocks

Constant ω	Lagged production ψ	Upstream effects β_{up}	Downstream effects β_{down}
-502.09	1.01	7.64	19.58
(70.05)	(<0.01)	(5.48)	(6.28)

Notes: Regression of sectoral output on money supply shocks using the Acemoglu et al. (2016) upstream/downstream decomposition presented in equation 14. The upstream and downstream variables are obtained by summing the unidirectional elements of the sum of the 2018 Leontief inverses. Standard errors are reported in parentheses. The second standard error is 3.2161×10^{-4}.

These empirical results have multiple implications. First, knowing the extent to which indirect effects are responsible for the propagation of monetary shocks (table 3) allows to better forecast the impact of monetary interventions. Moreover, providing the sectoral network effects estimates for each sector (table 2) could further improve forecasts if one were to know which sectors would initially be the most directly impacted. Second, knowing that capital linkages account for a fifth of these results hints that not including them in further research would underestimate the effects of the network structure of production. Lastly, the observation that downstream effects are much larger than upstream effects (table 4) implies that the monetary transmission channels focused on firms and investments (Tobin's Q channel, balance sheet channel, investment effect of the interest rate channel...) are more affected by the network structure of the economy than channels focused on households and consumption (household wealth effects, household liquidity effects, durable expenditure effect of the interest rate channel...). This provides further understanding of how the investment decisions of firms influences the decision of other firms, propagating monetary shocks with their materials and capital linkages.

5 Conclusion

Modelling monetary transmission in the production network setup is challenging due to the variety of transmission channels. By focusing on capital accumulation and investments dynamics through the interest rate channel and asset price channel, the modified model allows for a bidirectional propagation pattern of money supply shocks that propagate both upstream and downstream. Empirically, network effects differ significantly between sectors but explain a large part of how money supply shocks propagate and are amplified through intersectoral trade.

This analysis can be improved as follows. Theoretically, the model needs to be adapted with a CES production function or with sticky prices in order to match empirical data better. Moreover, using the nested CES production function of Baqaee and Farhi (2018) would allow for more complex propagation patterns than the upstream/downstream effects theorized in this paper. Empirically, the causal identification can be improved as many similar macro networks have been criticized for their lack of a clean identification. This could be achieved by using firm-level data instead of sector aggregates, where individual technology shocks could be instrumentalised to better estimate the impact of monetary shocks. While these datasets are rare, this analysis could be based on the Japanese firm-level credit reporting data, as done by Carvalho et al. (2021).

Overall, network effects and input-output linkages are shown to be a critical part of the propagation of monetary shocks, so that central banks must take them into consideration in their forecasts and monetary policy decisions.

Appendix

A Social planner Lagrangian

The Lagrangian of the social planner problem is as follows, since the non-negativity constraints are slack to avoid infinite marginal productivities:

$$\mathscr{L}_i = \mathbb{E}_0 \sum_{t=0}^{\infty} \left[\beta^t \sum_{i=1}^{n} \eta_i \log\left(\frac{c_{it}}{\eta_i}\right) \right] + \sum_{i,t} \lambda_{it} \left[z_{it} \zeta_i l_{it}^{\alpha_i} k_{it}^{\gamma_i} \prod_{j=1}^{n} x_{ijt}^{a_{ij}} - c_{it} - \sum_{j=1}^{n} x_{jit} - \sum_{j=1}^{n} m_{jit} \right]$$
$$+ \sum_{i,t} \mu_{it} \left[(1-\delta) k_{it} + q_{it} \prod_{j=1}^{n} m_{ijt}^{\theta_{ij}} - k_{i,t+1} \right] + \sum_t \phi_t \left[1 - \sum_i l_{it} \right]$$

B Derivation of the capital FOC

Let σ_{it} be the shadow value of the capital accumulation constraint for firm i in time t. The non-negativity constraints are slack as infinite marginal productivities would contradict an optimum. The firm problem Lagrangian is:

$$\mathscr{L}_i = \mathbb{E}_0 \sum_{t=0}^{\infty} \left[p_{it} y_{it} - w_t l_{it} - \sum_{j=1}^{n} p_{jt}(x_{ijt} + m_{ijt}) \right] + \sum_{t=1}^{\infty} \sigma_{it} \left[(1-\delta) k_{it} + q_{it} \prod_{j=1}^{n} m_{ijt} - k_{i,t+1} \right]$$

The first order condition with respect to m_{ijt} and $k_{i,t+1}$ are:

$$p_{jt} = \sigma_{it} q_{it} \theta_{ij} \frac{\prod_{j=1}^{n} m_{ijt}^{\theta_{ij}}}{m_{ijt}}$$
$$\sigma_{it} = (1-\delta)\sigma_{i,t+1} + p_{i,t+1} \gamma_i \frac{y_{i,t+1}}{k_{i,t+1}}$$

The first condition gives a system of n equations for each firm i that can be solve for the n intermediate input m_{ijt}. For instance if $n = 3$, there are 3 equations with 3 unknowns $m_{11t}, m_{12t}, m_{13t}$ (which includes self-loops). Solving this system for n sectors and using the CRS assumption $\sum_j \theta_{ij} = 1$ gives:

$$\sigma_{it} = \frac{1}{q_{it}} \prod_{j=1}^{n} \left(\frac{p_{jt}}{\theta_{ij}} \right)^{\theta_{ij}}$$

Substituting it in the first order condition for $k_{i,t+1}$ and solving for it gives:

$$k_{i,t+1} = p_{i,t+1} \gamma_i y_{i,t+1} \frac{q_{it} q_{i,t+1} \prod_j \theta_{ij}^{\theta_{ij}}}{(1-\delta) q_{i,t+1} \prod_j p_{jt}^{\theta_{ij}} - q_{it} \prod_j p_{j,t+1}^{\theta_{ij}}}$$

C Proof of theorem 1

Theorem 1 (Upstream propagation). *Let L^m be the Leontief inverse of the material linkages and Θ be the the capital linkages. The total output is:*

$$\sum_{i=1}^{n} p_{it} y_{it} = M_t + \sum_{i=1}^{n}\sum_{j=1}^{n} p_{jt} a_{ji} y_{jt} + \sum_{i=1}^{n}\sum_{j=1}^{n} p_{j,t+1} \gamma_j \theta_{ji} y_{j,t+1} \frac{q(\pi_{jt}, M_t) v_{jt}}{k_{j,t+1}}$$

And the output of all industries is:

$$\begin{bmatrix} p_{1t} y_{1t} \\ \vdots \\ p_{nt} y_{nt} \end{bmatrix} = L^{m'} \begin{bmatrix} p_{1t} c_{1t} \\ \vdots \\ p_{nt} c_{nt} \end{bmatrix} + L^{m'} \Theta' \left(\begin{bmatrix} p_{1,t+1} y_{1,t+1} \\ \vdots \\ p_{n,t+1} y_{n,t+1} \end{bmatrix} \circ \begin{bmatrix} \gamma_1 q(\pi_{1t}, M_t) v_{1t} / k_{1,t+1} \\ \vdots \\ \gamma_n q(\pi_{nt}, M_t) v_{nt} / k_{n,t+1} \end{bmatrix} \right)$$

Hence, money supply shocks propagate upstream.

Proof. Substituting the industries first order conditions (eq. 8, 9, 10) into the market clearing condition of industry i (eq. 7) gives:

$$y_{it} = c_{it} + \sum_{j=1}^{n} \frac{p_{jt}}{p_{it}} a_{ji} y_{jt} + \sum_{j=1}^{n} \frac{p_{j,t+1}}{p_{it}} \gamma_j \theta_{ji} y_{j,t+1} \frac{q_{jt} v_{jt}}{k_{j,t+1}}$$

Multiplying by p_{it}, summing over all industries, and using the cash-in-advance assumption ($M_t \equiv \sum_{j=1}^{n} p_{jt} c_{jt}$) gives:

$$\sum_{i=1}^{n} p_{it} y_{it} = M_t + \sum_{i=1}^{n}\sum_{j=1}^{n} p_{jt} a_{ji} y_{jt} + \sum_{i=1}^{n}\sum_{j=1}^{n} p_{j,t+1} \gamma_j \theta_{ji} y_{j,t+1} \frac{q_{jt} v_{jt}}{k_{j,t+1}}$$

Which is the first result. Next, instead of summing all industries, the output can be put in a matrix form:

$$Y_t = C_t + A' Y_t + \Theta' (Y_{t+1} \circ Q_t)$$

$$\text{Where: } Y_t = \begin{bmatrix} p_{1t} y_{1t} \\ \vdots \\ p_{nt} y_{nt} \end{bmatrix}_{n\times 1} \quad C_t = \begin{bmatrix} p_{1t} c_{1t} \\ \vdots \\ p_{nt} c_{nt} \end{bmatrix}_{n\times 1} \quad Q_t = \begin{bmatrix} \gamma_1 q_{1t} v_{1t} / k_{1,t+1} \\ \vdots \\ \gamma_n q_{nt} v_{nt} / k_{n,t+1} \end{bmatrix}_{n\times 1}$$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}_{n\times n} \quad \Theta = \begin{bmatrix} \theta_{11} & \cdots & \theta_{1n} \\ \vdots & \ddots & \vdots \\ \theta_{n1} & \cdots & \theta_{nn} \end{bmatrix}_{n\times n}$$

And $\circ$ is the Hadamard product (element-wise multiplication). Solving for Y_t gives:

$$Y_t = (I - A')^{-1} C_t + (I - A')^{-1} \Theta' (Y_{t+1} \circ Q_t)$$

Noting that $(I-A')^{-1}$ is the transpose of the Leontief inverse gives:

$$Y_t = L^{m'} C_t + L^{m'} \Theta' (Y_{t+1} \circ Q_t)$$

Money supply shocks thus modify the industries output by their impact on $p_{it} c_{it}$. Because the production of each industry depends on the production of its clients, money supply shocks propagate upstream. □

D Proof of theorem 2

Theorem 2 (**Downstream propagation**). *Let L^m, L^k be the Leontief inverse of the material linkages and capital linkages. Relative prices are determined by the supply-side technology (z_{it}) and money supply via capital efficiency (q_{it}):*

$$\begin{bmatrix} \log(p_{1t}/w_t) \\ \vdots \\ \log(p_{nt}/w_t) \end{bmatrix} = -(I_n - A - \Theta)^{-1} \left(\begin{bmatrix} \log(z_1) \\ \vdots \\ \log(z_n) \end{bmatrix} + \begin{bmatrix} \gamma_1 \log(q_{1t} q_{1,t-1}) \\ \vdots \\ \gamma_n \log(q_{nt} q_{n,t-1}) \end{bmatrix} \right)$$

Hence, money supply shocks propagate downstream.

Proof. Substituting the industries first order conditions (eq. 8, 9, 11) in the industries production function (eq. 1) gives:

$$y_{it} = z_{it} \zeta_i \left(\frac{p_{it}}{w_t} \alpha_i y_{it} \right)^{\alpha_i} \left(p_{it} \gamma_i y_{it} \frac{q_{i,t-1} q_{i,t} \prod_j \theta_{ij}^{\theta_{ij}}}{(1-\delta) q_{it} \prod_j p_{j,t-1}^{\theta_{ij}} - q_{i,t-1} \prod_j p_{jt}^{\theta_{ij}}} \right)^{\gamma_i} \prod_{j=1}^{n} \left(\frac{p_{it}}{p_{jt}} a_{ij} y_{it} \right)^{a_{ij}}$$

Defining efficiency-weighted prices $\tilde{p}_{jt}^{\theta_{ij}} := (1-\delta) q_{it} \prod_j p_{j,t-1}^{\theta_{ij}} - q_{i,t-1} \prod_j p_{jt}^{\theta_{ij}}$ and using the normalisation constant $\zeta_i := \alpha_i^{-\alpha_i} \gamma_i^{-\gamma_i} \prod_j \left(a_{ij}^{-a_{ij}} \theta_{ij}^{-\gamma_i \theta_{ij}} \right)$, the equation simplifies to:

$$y_{it} = z_{it} \left(\frac{p_{it}}{w_t} \right)^{\alpha_i} y_{it}^{\alpha_i + \gamma_i + \sum a_{ij}} (q_{i,t-1} q_{it})^{\gamma_i} p_{it}^{\gamma_i} \prod_{j=1}^{n} \left(\frac{p_{it}}{p_{jt}} \right)^{a_{ij}} \left(\frac{1}{\tilde{p}_{jt}} \right)^{\theta_{ij} \gamma_i}$$

Using the constant returns to scale assumption $\alpha_i + \gamma_i + \sum_j a_{ij} = 1$ to simplify the y_{it} and taking the logs gives:

$$0 = \log(z_{it}) + \alpha_i \log\left(\frac{p_{it}}{w_t} \right) + \gamma_i \log(q_{i,t-1} q_{it}) + \sum_{j=1}^{n} a_{ij} \log\left(\frac{p_{it}}{p_{jt}} \right) + \gamma_i \sum_{j=1}^{n} \theta_{ij} \log\left(\frac{p_{it}}{\tilde{p}_{jt}} \right)$$

Rearranging the logs by combining the $\log(p_{it})$ and adding/subtracting $\log(w_t)$ gives:

$$\log\left(\frac{p_{it}}{w_t} \right) = \sum_{j=1}^{n} \theta_{ij} \log\left(\frac{\tilde{p}_{jt}}{w_t} \right) + \sum_{j=1}^{n} a_{ij} \log\left(\frac{p_{jt}}{w_t} \right) - \log(z_{it}) - \gamma_i \log(q_{i,t-1} q_{it})$$

This equation holds for all industries i. This gives a system of n equations that can be written in matrix form:

$$P_t = \Theta \tilde{P}_t + AP_t - (\epsilon_t + \psi_t)$$

Where: $P_t = \begin{bmatrix} \log(p_{1t}/w_t) \\ \vdots \\ \log(p_{nt}/w_t) \end{bmatrix}_{n\times 1}$ $\tilde{P}_t = \begin{bmatrix} \log(\tilde{p}_{1t}/w_t) \\ \vdots \\ \log(\tilde{p}_{nt}/w_t) \end{bmatrix}_{n\times 1}$ $\epsilon_t = \begin{bmatrix} \log(z_1) \\ \vdots \\ \log(z_n) \end{bmatrix}_{n\times 1}$ $\psi_t = \begin{bmatrix} \gamma_1 \log(q_{1t}q_{1,t-1}) \\ \vdots \\ \gamma_n \log(q_{nt}q_{n,t-1}) \end{bmatrix}_{n\times 1}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}_{n\times n} \quad \Theta = \begin{bmatrix} \theta_{11} & \cdots & \theta_{1n} \\ \vdots & \ddots & \vdots \\ \theta_{n1} & \cdots & \theta_{nn} \end{bmatrix}_{n\times n}$$

A common result in RBC networks models is that $p_{jt} = w_t \; \forall j$ (Carvalho and Tahbaz-Salehi, 2019). In the investment model, this must extend to $p_{jt} = w_t = \tilde{p}_{jt} \; \forall j$, otherwise the marginal productivity of capital would not be equal to its marginal costs, which contradicts the maximisation of profits. As a result, $P_t = \tilde{P}_t$. Solving for the prices gives:

$$P_t = -(I_n - A - \Theta)^{-1}(\epsilon_t + \psi_t)$$

Money supply shocks thus modify relative prices by their impact on ψ. Following Carvalho and Tahbaz-Salehi (2019), it implies that money supply shocks propagate downstream since the change of relative prices impacts the intermediate customers and household decisions. $\square$

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